3.15.1. Deductions and Proofs: Problems

A. For each of the following formal arguments, show that the argument is valid by constructing a **deduction** of the argument.

1. $((P \vee Q) \rightarrow R) \ \therefore \ ((P \vee Q) \rightarrow R)$

2. $((P \rightarrow Q) \rightarrow P) \ \therefore \ ((P \rightarrow Q) \rightarrow Q)$

3. $(P \rightarrow R) \ \therefore \ ((P \wedge Q) \rightarrow R)$

4. $(P \rightarrow Q)\ .\ (Q \rightarrow R) \ \therefore \ (P \rightarrow R)$

5. $((P \rightarrow Q) \rightarrow (R \rightarrow S))\ .\ (R \rightarrow T)\ .\ ((R \rightarrow T) \rightarrow (P \rightarrow Q))\ .$
$(Q \rightarrow (S \rightarrow U))\ .\ (T \rightarrow P) \ \therefore \ (R \rightarrow U)$

6. $(P \rightarrow \sim Q)\ .\ (R \rightarrow Q)\ .\ (\sim R \rightarrow \sim S)\ .\ (T \rightarrow S) \ \therefore \ (P \rightarrow \sim T)$

7. $(P \rightarrow R)\ .\ (Q \rightarrow R) \ \therefore \ ((P \vee Q) \rightarrow R)$

8. $(R \rightarrow (P \vee Q))\ .\ (Q \rightarrow P) \ \therefore \ (\sim P \rightarrow \sim R)$

9. $((P \vee Q) \rightarrow R)\ .\ (R \rightarrow S)\ .\ (T \rightarrow Q)\ .\ \sim S \ \therefore \ \sim T$

10. $(R \rightarrow (P \wedge Q)) \ \therefore \ (\sim P \rightarrow \sim R)$

11. $(R \rightarrow (P \vee Q))\ .\ (P \rightarrow S)\ .\ ((S \vee T) \rightarrow Q)\ .\ \sim Q \ \therefore \ \sim(R \vee T)$

12. $((P \wedge Q) \rightarrow R)\ .\ (\sim S \rightarrow (Q \wedge \sim R)) \ \therefore \ (P \rightarrow S)$

B. **Translate** each of the following English arguments into the formal language of Chapter Three, then show that the argument is valid by constructing a **deduction** of it.

1. If Jake's at the Bel-Aire, then if he's at the Bel-Aire he's playing darts. ∴ If Jake's at the Bel-Aire he's playing darts.

2. If Kitty won the hand only if she drew a 7 Bam then she won the hand. ∴ Kitty won the hand.[1]

3. Either Dick or Dora is having a Gibson. If either of them is having a Gibson, Dick is. Dick is having a Gibson only if Dora is. ∴ Both Dick and Dora are having a Gibson.

4. Assuming Trixie passed Logic if the test wasn't too hard, Barbie passed Logic. If Trixie didn't pass Logic, then the test was too hard. ∴ Barbie passed Logic.

5. If the bartender is the killer then Dick will catch her in a lie, assuming Dora joins the conversation. Provided that Dick will catch the bartender in a lie if the bartender is the killer, the bartender will confess to the crime. The bartender will confess to the crime only if she's the killer. ∴ If Dora joins the conversation Dick will catch the bartender in a lie.

6. If Letitia's going to the party then Lucretia's not going. Letitia's going to the party if and only if Lucretia is. ∴ Neither Letitia nor Lucretia are going to the party.

7. If Kitty's getting a manicure, then she'll have a massage only if the check cleared. The check didn't clear, but Kitty's getting a manicure. ∴ Kitty won't have a massage.

(Can be done without ID if DM is used.)

8. Kitty will have both a manicure and a massage if the check cleared, and she'll have a manicure without (having) a massage otherwise. ∴ Kitty will have a manicure, and she'll have a massage if and only if the check cleared.

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[1] Based on Peirce's Law, set out in (Peirce 1884)

9. Rex is making a tuna sandwich if Neko's busy working on her invention, and a seafood casserole otherwise. Neko's busy working on her invention only if Rex is making a seafood casserole. ∴ Rex is making a seafood casserole.

10. Rex is making a tuna sandwich if Neko's busy working on her invention, and a seafood casserole otherwise. Neko's busy working on her invention if and only if Rex is making a seafood casserole. ∴ Neko's busy working on her invention and Rex is making a seafood casserole.

11. That consonantal segment is prevocalic if it occurs initially; otherwise it's voiceless. Provided it's either prevocalic or voiceless, it's both continuant and strident. Assuming it's continuant, it's tense if it's strident. If it's tense, then if it occurs initially it's palatalized. ∴ That consonantal segment is palatalized and voiceless.

(*Adapted from* Partee, ter Meulen and Wall 1990: 134, Problem 10e)

12. The president will sign an executive order if the bill stalls in either the House or the Senate. The bowling lobby will mobilize only if the bill stalls in the Senate. Assuming Pachinko PAC holds a phone campaign, the bill will stall in the House. If Pachinko PAC doesn't hold a phone campaign, the bowling lobby will mobilize. Therefore, the president will sign an executive order.

13. If the bartender didn't kill the baron, then either the sommelier or the bootlegger did. The merlot was poisoned if the sommelier killed the baron. There was antifreeze in the sour mash if the bootlegger killed the baron. The merlot wasn't poisoned, and the bartender didn't kill the baron. ∴ The bootlegger killed the baron

(*Adapted from* Partee, ter Meulen and Wall 1990: 134, Problem 10a)

14. Either Neko is a cat who can't stop eating, or Jack is a cat who's been stealing Neko's food. Neko can stop eating if Jack hasn't been stealing her food. Neko is a cat if and only if Jack is. Therefore, Jack is a cat who's been stealing Neko's food.

15. If God exists then S/he's omnipotent. If God exists then S/he's omniscient. If God exists then S/he's benevolent. If God can prevent evil, then if S/he knows that evil exists, then S/he's not benevolent if S/he doesn't prevent it. If God is omnipotent then S/he can prevent evil. If God is omniscient, then S/he knows that evil exists if it does indeed exist. Evil doesn't exist if God prevents it. Evil exists. Therefore, God doesn't exist.

(*Adapted from* Kalish, Montague, and Mar 1980: 35, Problem 35)

C. Show that each of the following sentences is a **theorem**, by constructing a **proof** of that sentence.

T3.1. $(P \rightarrow P)$

T3.2. $((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R)))$

T3.3a. $(P \rightarrow (\sim P \rightarrow Q))$
T3.3b. $(\sim P \rightarrow (P \rightarrow Q))$

T3.4. $(P \rightarrow ((P \rightarrow Q) \rightarrow Q))$

T 3.5. $(((P \rightarrow Q) \rightarrow P) \leftrightarrow P)$
    T 3.5a. $(((P \rightarrow Q) \rightarrow P) \rightarrow P)$
    T 3.5b. $(P \rightarrow ((P \rightarrow Q) \rightarrow P))$

T 3.6. $(((P \rightarrow Q) \leftrightarrow (P \rightarrow (P \wedge Q))$
    T 3.6a. $((P \rightarrow Q) \rightarrow (P \rightarrow (P \wedge Q)))$
    T 3.6b. $((P \rightarrow (P \wedge Q)) \rightarrow (P \rightarrow Q))$

T3.7. $((P \rightarrow Q) \vee (Q \rightarrow P))$

T3.8. $((P \rightarrow Q) \rightarrow ((P \vee R) \rightarrow (Q \vee R)))$

T9. $(((P \rightarrow Q) \wedge (P \rightarrow R)) \leftrightarrow (P \rightarrow (Q \wedge R)))$
    T9a. $(((P \rightarrow Q) \wedge (P \rightarrow R)) \rightarrow (P \rightarrow (Q \wedge R)))$
    T9b. $((P \rightarrow (Q \wedge R)) \rightarrow ((P \rightarrow Q) \wedge (P \rightarrow R)))$

T3.10a. $(\sim P \rightarrow \sim(P \wedge Q))$
T3.10b. $(\sim Q \rightarrow \sim(P \wedge Q))$

T3.11. $(((P \rightarrow Q) \wedge (R \rightarrow S)) \rightarrow ((P \wedge R) \rightarrow (Q \wedge S)))$

T3.12. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \wedge Q) \rightarrow R))$

T3.12a. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.12b. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.13. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow (Q \rightarrow (P \rightarrow R)))$

T3.13a. $((P \rightarrow (Q \rightarrow R)) \rightarrow (Q \rightarrow (P \rightarrow R)))$

T3.13b. $((Q \rightarrow (P \rightarrow R)) \rightarrow (P \rightarrow (Q \rightarrow R)))$

T3.14. $((P \rightarrow \sim P) \leftrightarrow \sim P)$

T3.14a. $((P \rightarrow \sim P) \rightarrow \sim P)$

T3.14b. $(\sim P \rightarrow (P \rightarrow \sim P))$

T3.15. $((P \rightarrow (Q \wedge \sim Q)) \leftrightarrow \sim P)$

T3.15a. $((P \rightarrow (Q \wedge \sim Q)) \rightarrow \sim P)$

T3.15b. $(\sim P \rightarrow (P \rightarrow (Q \wedge \sim Q)))$

T3.16. $((P \rightarrow Q) \leftrightarrow (\sim P \vee Q))$

T3.16a. $((P \rightarrow Q) \rightarrow (\sim P \vee Q))$

T3.16b. $((\sim P \vee Q) \rightarrow (P \rightarrow Q))$

T3.17. $((P \rightarrow Q) \leftrightarrow \sim(P \wedge \sim Q))$

T3.17a. $((P \rightarrow Q) \rightarrow \sim(P \wedge \sim Q))$

T3.17b. $(\sim(P \wedge \sim Q) \rightarrow (P \rightarrow Q))$

T3.18. $((P \wedge P) \leftrightarrow P)$

T3.18a. $((P \wedge P) \rightarrow P)$

T3.18b. $(P \rightarrow (P \wedge Q))$

T3.19. $((P \vee P) \leftrightarrow P)$

T3.19a. $((P \vee P) \rightarrow P)$

T3.19b. $(P \rightarrow (P \vee Q))$

T3.20. $(P \leftrightarrow P)$

T3.21a. $\sim(P \leftrightarrow \sim P)$

T3.21b. $\sim(\sim P \leftrightarrow P)$

T3.22. $(\sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q))$

T3.22a. $(\sim(P \leftrightarrow Q) \rightarrow (P \leftrightarrow \sim Q))$

T3.22b. $((P \leftrightarrow \sim Q) \rightarrow \sim(P \leftrightarrow Q))$

T3.23. $((P \leftrightarrow Q) \leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)))$

T3.23a. $((P \leftrightarrow Q) \rightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)))$

T3.23b. $(((P \rightarrow Q) \wedge (Q \rightarrow P)) \rightarrow (P \leftrightarrow Q))$

T3.24. $((P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)))$

T3.24a. $((P \leftrightarrow Q) \rightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)))$

T3.24b. $(((P \wedge Q) \vee (\sim P \wedge \sim Q)) \rightarrow (P \leftrightarrow Q))$

E. It was noted in 3.6 that every argument in the formal language has a corresponding **leading principle**: a conditional whose antecedent is the conjunction of that argument's premises, and whose consequent is the conclusion of the argument. So the following argument has the leading principle listed below.

1. If Rex's team lost, then Rex is upset.	1. $(P \rightarrow Q)$
2. Rex's team lost.	2. P
$\therefore$ Rex is upset.	$\therefore Q$

Leading Principle: $(\underline{(P \rightarrow Q) \wedge P}) \rightarrow Q)$

Armed now with conditional deduction and Modus Ponens, show that **if an argument's leading principle is a theorem** (capable of a proof appealing to no premises), then the **argument is valid** (so: there's a deduction of that argument's conclusion from its premises).